It is necessary to make the units compatible on both sides of the relationship. Let us multiply the BM units by 10^3 to convert the kilonewtons to newtons, and by a further 10^3 to convert metres to millimetres. Then

$$\frac{7 \times 50 \times d^2}{6} = \frac{4 \times 10^3 \times 4.5 \times 10^3}{8}$$

$$d^2 = \frac{4 \times 4.5 \times 10^6 \times 6}{8 \times 7 \times 50}$$

$$d = \sqrt{\left(\frac{4 \times 4.5 \times 10^6 \times 6}{8 \times 7 \times 50}\right)} = 196.4\,\text{mm}$$

Use a 50 mm × 200 mm timber beam.

Example 1.7

Calculate the depth required for the timber beam shown in Figure 1.17a if the breadth is 75 mm and the permissible bending stress is 8.5 N/mm². An allowance for the self-weight of the beam has been included with the point loads.

We have $b = 75\,\text{mm}$ and $f = 8.5\,\text{N/mm}^2$; d is to be found. To complete the load diagram it is first necessary to calculate the reactions. Take moments about end B, clockwise moments being positive and anti-clockwise moments negative:

$$8R_a = (3 \times 6) + (5 \times 2)$$
$$8R_a = 18 + 10$$
$$8R_a = 28$$
$$R_a = 28/8 = 3.5\,\text{kN}$$

Therefore $R_b = 8 - 3.5 = 4.5\,\text{kN}$.

Having calculated the reactions to complete the load diagram, the shear force diagram Figure 1.17b can be constructed. This shows that a point of contraflexure occurs under the 5 kN point load, and hence the maximum bending moment will be developed at that position. The bending moment diagram for the beam is shown in Figure 1.17c.

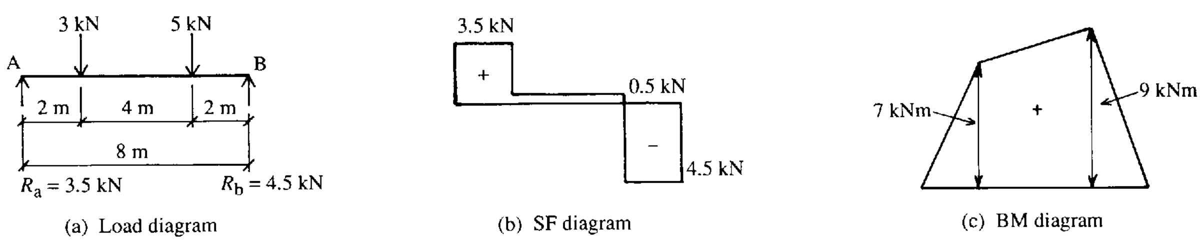

Figure 1.17 *Timber beam diagrams*

BM under 3 kN point lead $= 3.5 \times 2 = 7\,\text{kN m} = 7 \times 10^6\,\text{N mm}$

Maximum BM under 5 kN point load $= 4.5 \times 2 = 9\,\text{kN m} = 9 \times 10^6\,\text{N mm}$

The maximum BM is equated to the internal moment of resistance:

$$\text{Internal MR} = \text{external BM maximum}$$

$$f\frac{bd^2}{6} = 9 \times 10^6$$

$$\frac{8.5 \times 75 \times d^2}{6} = 9 \times 10^6$$

$$d = \sqrt{\left(\frac{9 \times 10^6 \times 6}{8.5 \times 75}\right)} = 291\,\text{mm}$$

Use a 75 mm × 300 mm timber beam.

Example 1.8

A steel beam supports a total UDL including its self-weight of 65 kN over a span of 5 m. If the permissible bending stress for this beam is taken as 165 N/mm^2, determine the elastic modulus needed for the beam.

We have

$$\text{Internal MR} = \text{external BM maximum}$$

$$fZ = \frac{WL}{8}$$

$$165Z = \frac{65 \times 10^3 \times 5 \times 10^3}{8}$$

$$Z = \frac{65 \times 5 \times 10^6}{8 \times 165}$$

$$Z = 246\,212\,\text{mm}^3 = 246.21\,\text{cm}^3$$

Therefore the elastic modulus Z needed for the beam is 246.21 cm^3. Section property tables for steel beams give the elastic modulus values in cm^3 units. By reference to such tables we see that a 254 mm × 102 mm × 25 kg/m universal beam section, which has an elastic modulus of 265 cm^3, would be suitable in this instance.

Example 1.9

A timber beam spanning 5 m supports a UDL of 4 kN which includes an allowance for its self-weight. If a 100 mm wide by 200 mm deep beam is used, calculate the bending stress induced in the timber. What amount of deflection will be produced by the load if the E value for the timber is 6600 N/mm^2, and how does this compare with a permissible limit of 0.003 × span?

We know $b = 100$ mm and $d = 200$ mm; f is to be found. We have

$$\text{Internal MR} = \text{external BM maximum}$$

$$f\frac{bd^2}{6} = \frac{WL}{8}$$

$$\frac{f \times 100 \times 200^2}{6} = \frac{4 \times 10^3 \times 5 \times 10^3}{8}$$

$$f = \frac{4 \times 5 \times 10^6 \times 6}{8 \times 100 \times 200^2} = 3.75\,\text{N/mm}^2$$